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# Experimental and computational investigation of interfacial shear along a wavy two-phase interface

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ABSTRACT

This study explores the complex fluid flow behavior adjacent to the interface between parallel layers of gas and liquid. Using water and nitrogen as working fluids, the interface is examined experimentally using high-speed video, and the flow structure predicted using FLUENT. The computational model is used to analyze the gas flow near the interface by isolating and examining a domain that represents an instantaneous snapshot of the wavy interface. Both the observed and computed interfaces show appreciable interfacial waviness, which increases in intensity with increasing flow rates; they also show gas entrainment effects at high flow rates. The computed results show turbulence is completely suppressed along the interface by surface tension. Computed velocity vector plots, contour plots and flow streamlines show interfacial flow separation on the gas side, and these effects are amplified with increasing gas Reynolds number. This produces form drag along the wavy interface in addition to the viscous drag. The interfacial viscous and form drag components increase monotonically with increasing ratio of wave height to wavelength because of the increased frictional resistance and flow separation effects, respectively. A new relation for the interfacial friction factor is derived from the computational results, which agrees well with prior turbulent flow correlations.

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## 1. Introduction

Two-phase flow models incorporate several transport parameters that are represented in terms of fluid properties, flow rates and length scales. However, these models are further complicated by interfacial traits that are not easily predicted. The interfacial wave structure and interfacial dampening of turbulent eddies have been the focus of considerable study [1–3]. These, in turn, influence interfacial velocity and temperature gradient, key ingredients in the development of relations for interfacial mass transfer, shear and heat flux found in theoretical two-phase flow models. The dynamics of fluid flow along turbulent interfaces needs to be investigated in order to resolve the inter-dependent nature of these interfacial parameters.

Interfacial shear can be neglected in the case of free-falling films [4,5]. Where interfacial shear is significant, empirical expressions are incorporated into two-phase models with varying degrees of difficulty, such as the homogeneous equilibrium model and slip-flow model. Empirical relations for interfacial shear aim to account for interfacial momentum transfer due to evaporation or condensation [6], as well as interfacial waviness [6–8]. The pursuit of an improved model for interfacial shear requires systematic assessment of key transport parameters of a turbulent wavy interface, such as both shear and drag forces, eddy diffusivity and length scales associated with a wavy interface.

### 1.1. Interfacial drag

An examination of literature on the fluid dynamics of two-phase flows shows a far greater focus on interfacial shear as compared to interfacial drag. Ishii and Zuber [9] constructed a unified law for drag coefficient in dispersed flows. Kataoka et al. [10] developed an expression for the interfacial drag coefficient in annular flow, which they used to predict droplet entrainment parameters. Because of the large differences between gas and liquid velocities in annular flow, it is useful to examine drag effects for gas flow along a wavy solid surface. Salvetti et al. [11] studied drag forces exerted along solid sinusoidal surfaces, and consolidated measured and simulated findings from previous studies. Their parametric study considered flow rate, fluid properties and surface profile as salient variables that influence the drag coefficient.

### 1.2. Interfacial shear in annular two-phase flow

Hartley and Roberts [12] were among the first investigators to recommend a relationship between interfacial friction coefficient,

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**Nomenclature**

| | |
|---|---|
| $A$ | local projected interfacial area |
| $C_D$ | combined (viscous plus form) drag coefficient |
| $C_{f,D}$ | coefficient of viscous drag |
| $C_{f,i}$ | interfacial skin friction coefficient |
| $C_p'$ | canonical pressure distribution |
| $C_{\varepsilon 1}$, $C_{\varepsilon 2}$ | constants in turbulent kinetic energy transport equation |
| $C_\mu$ | constants in Boussinesq equation |
| $D$ | hydraulic diameter of entire channel |
| $D_H$ | hydraulic diameter of liquid layer |
| $\hat{e}_d$ | unit vector parallel to flow direction |
| $F_{D,form}$ | form drag |
| $F_{D,visv}$ | viscous drag |
| $f_g$ | wall friction factor for Domain 2 |
| $f_i$ | interfacial friction factor |
| $g$ | gravitational acceleration |
| $h$ | interfacial wave height |
| $k$ | turbulent kinetic energy, constant in Stratford [43] separation theory |
| $l$ | interfacial wavelength |
| $N$ | number of discrete data points |
| $n$ | number of samples in subset of data record |
| $\hat{n}$ | unit vector normal to interface |
| $P$ | pressure; probability |
| $p$ | probability density |
| $Re$ | Reynolds number |
| $S$ | area of curved interface |
| $t$ | time |
| $\hat{t}$ | unit vector parallel to interface |
| $U$ | inlet mean $x$-direction velocity |
| $u_x$ | $x$-direction velocity component |
| $\bar{u}_{x,m}$ | local $x$-direction velocity component averaged over $y$-span at same location |
| $u_y$ | $y$-direction velocity component |
| $u_z$ | $z$-direction velocity component |
| $V$ | inlet $y$-direction velocity |
| $W$ | inlet $z$-direction velocity |
| $x, y, z$ | spatial coordinates |
| $x'$ | effective boundary layer length |

*Greek symbols*

| | |
|---|---|
| $\delta$ | annular film thickness |
| $\varepsilon$ | dissipation rate of turbulent kinetic energy |
| $\varepsilon_m$ | eddy momentum diffusivity |
| $\mu$ | dynamic viscosity |
| $\nu$ | kinematic viscosity |
| $\omega$ | turbulent specific dissipation |
| $\rho$ | density |
| $\sigma_\kappa$ | constant in turbulent kinetic energy transport equation |
| $\sigma_\varepsilon$ | constant in turbulent dissipation transport equation |
| $\tau$ | shear stress |

*Subscripts*

| | |
|---|---|
| *form* | form or pressure (drag) |
| *g* | gas |
| *i* | interface |
| *l* | liquid, laminar |
| *max* | maximum |
| *t* | turbulent |
| *visc* | viscous (drag) |
| *w* | wall |

*Superscripts*

| | |
|---|---|
| $-$ | mean component; average |
| + | non-dimensional |
| $'$ | fluctuating component |

$f_i$, and dimensionless film thickness in annular two-phase flow. Wallis [6] developed a theoretical model for interfacial shear, $\tau_i$, in annular flow in terms of flow rate and fluid properties. His model incorporated the influence of drag forces by modifying an expression by Silver and Wallis [13] based on the Reynolds flux concept. Wallis [14,15] later published at a curve fit for $f_i$ that yielded good agreement with pressure drop data for annular flow. Henstock and Hanratty [16] studied air–water annular flow assuming a known entrainment rate, and correlated interfacial shear to the mass flow rate. Using an extensive experimental database, Kataoka et al. [10] updated the Wallis correlation [14] to account for interfacial wave amplitude, which they expressed in terms of $f_i$ and fluid properties. Asali et al. [17] improved this correlation for vertical annular flows by employing an updated technique for measuring annular film thickness in the presence of droplet entrainment.

Narain et al. [18] studied annular condensing flows at different inclinations and proposed an asymptotic model for $\tau_i$ that showed good agreement with data. Other published models for $\tau_i$ include those of Mickley [19], Shekriladze and Gomelauri [20], Moeck [21], Andreussi [22], Soliman et al. [23], and Spedding and Hand [24]. Fukano and Furukawa [25] investigated the influence of liquid viscosity on interfacial shear in vertical annular upflow, and recommended a correlation for $\tau_i$ in terms of $f_i$ and fluid properties. Their unconventional formulation involved higher order terms that accounted for the significant increase in interfacial drag for large film thicknesses. Fore et al. [26] performed experiments involving vertical annular concurrent flow of water and nitrogen to broaden the application range of the Wallis [14] correlation. Their expression demarcated the behavior of $f_i$ at medium and high flow rates, as opposed to the uniform treatment of Wallis [15]. Using a large database and focusing on thick annular films, Wongwises and Kongkiatwanitch [27] recommended yet another correlation for $f_i$, which accounted for roughness effects over a wider range of flow rates. A common thread observed in all these models is their inability to demarcate the entrainment effects due to phase change from those due to fluid dynamics.

The present study concerns the interfacial characteristics and fluid dynamics of adiabatic horizontal flow of a water film that is shear driven by a nitrogen stream. A facility is constructed to generate a nearly two-dimensional water film whose interface could be captured using high-speed video imaging. Using FLUENT, a computational model is constructed to generate interfacial shape (profile), whose wavy features and turbulence effects are carefully examined. The liquid velocity and interfacial shear stress are inspected to gain detailed insight into their three-dimensional distributions in terms interfacial profile. Skin friction distribution and gas flow separation effects are also examined. A relation is derived for the interfacial friction factor as a function of both liquid film thickness and ratio of wave height to wavelength. Also proposed are relations for the drag coefficient, which can be used to derive criteria for droplet entrainment.

## 2. Experimental facility

An experimental facility is constructed to study the interfacial structure of horizontal adiabatic water–nitrogen flow. The facility

is comprised of a fluid delivery system, test section and video imaging system. Fig. 1(a) shows a schematic of the fluid delivery system, which is designed to deliver de-ionized water and nitrogen gas to the test module at desired flow rates. The fluid delivery system consists of a water flow loop and a nitrogen gas flow loop. In the water loop, de-ionized water from a reservoir is pumped through a 5-μm filter followed by one of several flow meters connected in parallel, before entering the test section. In the nitrogen loop, nitrogen gas is supplied from a pressure cylinder and measured by one of several gas flow meters. The water merges with the nitrogen in the test section, and the mixture is returned to the reservoir, where the nitrogen is vented to the ambient.

Fig. 1(b) and (c) show, respectively, an exploded view and photo of the test section. This main component of the experimental facility consists of a transparent flow module consisting of a base plate, top plate and cover plate, all made from transparent acrylic to facilitate optical and visual access to the flow within the channel. The channel itself is a 6-mm wide and 2.5-mm deep slot that is machined into the lower base plate. The base plate contains deep plenums leading to shallow plenums at both channel ends. A flow divider is inserted in the inlet plenum and is designed to transport water and nitrogen from the inlet ports to the channel with minimum swirl. Made from Accura 60 resin, the divider is fabricated using a rapid prototyping process. The divider is fitted with a 0.1-mm aluminum precision machined barrier plate, the downstream end of which is sharpened gradually to ensure smooth merging of the two fluids. The 39-mm length of the barrier plate ensures that the flows of both fluids overcome any upstream swirl for all flow rates considered, leaving a downstream channel length of 81 mm for interaction between the two fluids. The rectangular flow channel is closed atop by the 1.6-mm thick channel cover plate. The entire assembly, including the base plate, top plate, channel cover plate, and flow divider, are pressed together and held firmly in place with the aid of two sets of aluminum clamps. The cover plate has a slot on its underside that acts as a seat to accommodate the barrier plate. Not shown in Fig. 1(b) is an O-ring that is pressed in a groove in the base plate to guard against any leaks between the base plate and channel cover plate. The test section is oriented to ensure that the nitrogen gas flows over the water. Fig. 1(d) shows a photo of the complete test facility.

Experiments are performed for liquid Reynolds numbers ranging from $Re_l = 400$ to 11,000 to cover both laminar and turbulent liquid conditions. The liquid–gas interface is captured with the aid of a Photron FASTCAM-Ultima high-speed video camera fitted with a Micro-Nikkor 105-mm f/2.8G lens. Video segments are recorded for up to 0.682 s at 500 frames per second (fps) with a pixel resolution of 1024 × 256. Lighting is provided by 16 of 5-W white LEDs that are fitted into a fan-cooled enclosure.

## 3. Computational methods

The fluid flow system considered is three-dimensional, unsteady, turbulent and adiabatic. It is modeled by solving the

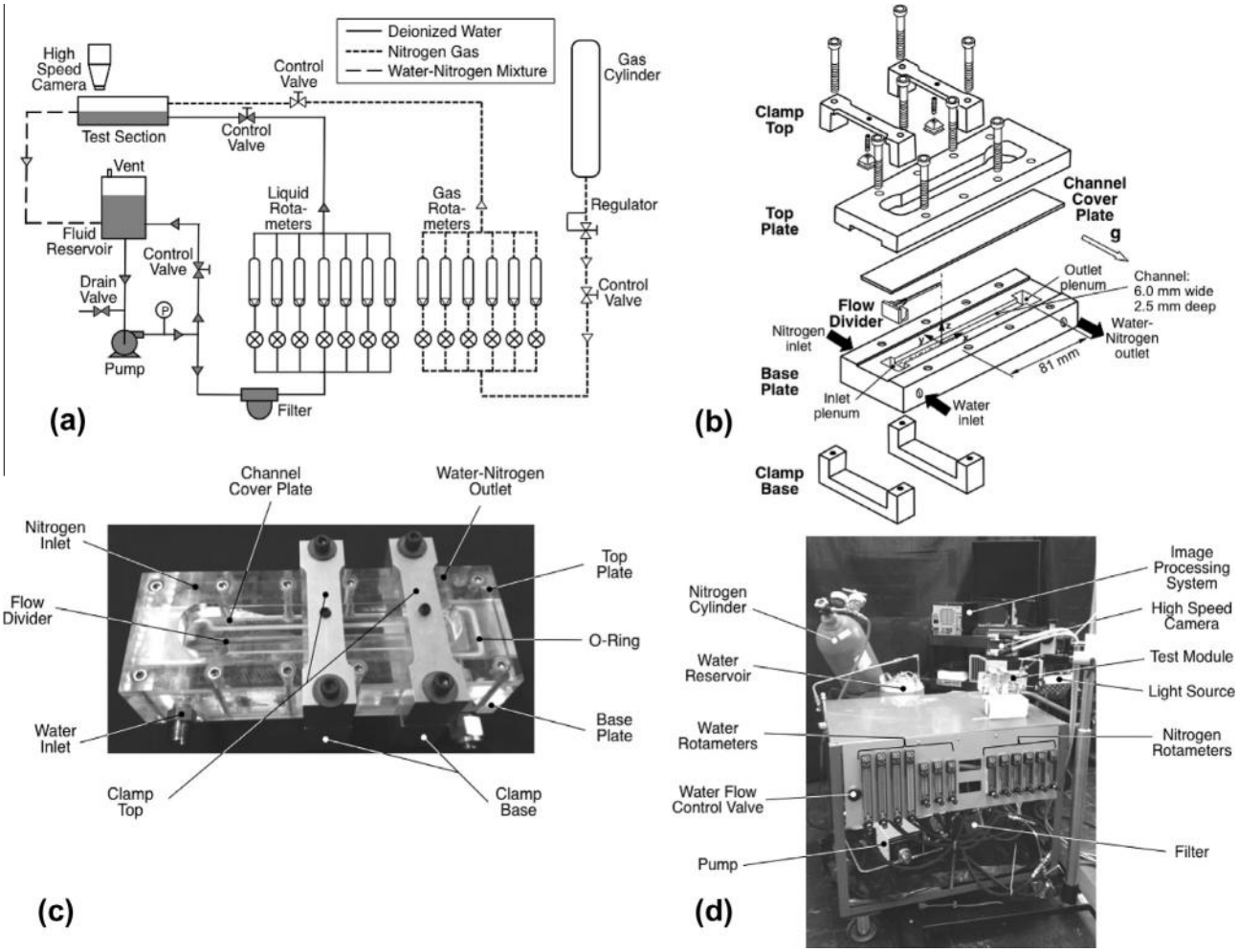


**Fig. 1.** (a) Schematic diagram of flow loop. (b) Exploded view of test section. (c) Photo of test section. (d) Photo of test facility.

three-dimensional unsteady Navier–Stokes equations in Cartesian coordinates. The mass and momentum equations are subjected to Reynolds decomposition [28], and time-averaged as [29]

$$\frac{\partial \bar{u}_x}{\partial x}+\frac{\partial \bar{u}_y}{\partial y}+\frac{\partial \bar{u}_z}{\partial z}=0, \tag{1}$$

$$\rho\left(\frac{\partial \bar{u}_x}{\partial t}+\bar{u}_x\frac{\partial \bar{u}_x}{\partial x}+\bar{u}_y\frac{\partial \bar{u}_x}{\partial y}+\bar{u}_z\frac{\partial \bar{u}_x}{\partial z}\right)=-\frac{\partial P}{\partial x}+\mu\left(\frac{\partial^2 \bar{u}_x}{\partial x^2}+\frac{\partial^2 \bar{u}_y}{\partial y^2}+\frac{\partial^2 \bar{u}_z}{\partial z^2}\right)+\rho\left(\frac{\partial}{\partial x}\left(-\langle u_x'^2\rangle\right)+\frac{\partial}{\partial x}\left(-\left\langle u_x'u_y'\right\rangle\right)+\frac{\partial}{\partial x}\left(-\langle u_x'u_z'\rangle\right)\right)+\rho g_x, \tag{2a}$$

$$\rho\left(\frac{\partial \bar{u}_y}{\partial t}+\bar{u}_x\frac{\partial \bar{u}_y}{\partial x}+\bar{u}_y\frac{\partial \bar{u}_y}{\partial y}+\bar{u}_z\frac{\partial \bar{u}_y}{\partial z}\right)=-\frac{\partial P}{\partial y}+\mu\left(\frac{\partial^2 \bar{u}_x}{\partial x^2}+\frac{\partial^2 \bar{u}_y}{\partial y^2}+\frac{\partial^2 \bar{u}_z}{\partial z^2}\right)+\rho\left(\frac{\partial}{\partial x}\left(-\langle u_y'u_x'\rangle\right)+\frac{\partial}{\partial y}\left(-\langle u_y'^2\rangle\right)+\frac{\partial}{\partial z}\left(-\langle u_y'u_z'\rangle\right)\right)+\rho g_y, \tag{2b}$$

and

$$\rho\left(\frac{\partial \bar{u}_z}{\partial t}+\bar{u}_x\frac{\partial \bar{u}_z}{\partial x}+\bar{u}_y\frac{\partial \bar{u}_z}{\partial y}+\bar{u}_z\frac{\partial \bar{u}_z}{\partial z}\right)=-\frac{\partial P}{\partial z}+\mu\left(\frac{\partial^2 \bar{u}_x}{\partial x^2}+\frac{\partial^2 \bar{u}_y}{\partial y^2}+\frac{\partial^2 \bar{u}_z}{\partial z^2}\right)+\rho\left(\frac{\partial}{\partial x}\left(-\langle u_z'u_x'\rangle\right)+\frac{\partial}{\partial y}\left(-\langle u_z'u_y'\rangle\right)+\frac{\partial}{\partial z}\left(-\langle u_z'^2\rangle\right)\right)+\rho g_z. \tag{2c}$$

Fig. 2 shows the two computational domains used in this study domains. Domain 1 represents the 81-mm long two-phase portion of the flow channel between the downstream edge of the barrier plate, where the two fluids begin to merge, to the downstream end of the channel. This domain allows the flow to establish a three-dimensional interface that is then exported to Domain 2. This second single-phase domain is used to explore the nitrogen flow alone along a wavy but stationary surface whose shape mimics that of the interface from Domain 1.

The boundary conditions for Domain 1 are specified as follows: at the inlet ($x = 0$), liquid velocity is assumed uniform and its magnitude based on the liquid Reynolds number, $U = \nu Re_1/D_H$, $V = 0$ and $W = 0$. The inlet velocity for nitrogen is prescribed in a similar manner, but adjusted to ensure that the interface is formed in the vicinity of $y = 3$ mm rather than shift towards the top or bottom walls of the channel. Fig. 3 shows, for each water Reynolds number, $Re_l$, the nitrogen Reynolds number, $Re_g$, that satisfies this interfacial criterion. Shown are clearly identifiable laminar and turbulent flow regimes based on the magnitude of $Re_l$. These regimes can be correlated according to the following relations.

$$\text{Laminar}: \quad \frac{\mathrm{Re}_g}{\mathrm{Re}_l}=14.42\mathrm{Re}_l^{-0.45}, \tag{3a}$$

$$\text{Turbulent}: \quad \frac{\mathrm{Re}_g}{\mathrm{Re}_l}=9.60\mathrm{Re}_l^{-0.31}. \tag{3b}$$

To resolve the turbulent terms in the governing equations, the $k$–$\varepsilon$ two-equation model [29] is used. The $k-\varepsilon$ model provides closure to the turbulent stress terms by employing additional transport equations for turbulent kinetic energy, $k$, and its dissipation rate, $\varepsilon$, given respectively, by

$$\frac{\partial k}{\partial t}+\frac{\partial k\bar{u}_x}{\partial x}+\frac{\partial k\bar{u}_y}{\partial y}+\frac{\partial k\bar{u}_z}{\partial z}=-\langle u_x'^2\rangle\frac{\partial \bar{u}_x}{\partial x}-\langle u_x'u_y'\rangle\frac{\partial \bar{u}_x}{\partial y}-\langle u_x'u_z'\rangle\frac{\partial \bar{u}_x}{\partial z}-\langle u_y'u_x'\rangle\frac{\partial \bar{u}_y}{\partial x}-\langle u_y'^2\rangle\frac{\partial \bar{u}_y}{\partial y}-\langle u_y'u_z'\rangle\frac{\partial \bar{u}_y}{\partial z}-\langle u_z'u_x'\rangle\frac{\partial \bar{u}_z}{\partial x}-\langle u_z'u_y'\rangle\frac{\partial \bar{u}_z}{\partial y}-\langle u_z'^2\rangle\frac{\partial \bar{u}_z}{\partial z}-\varepsilon+\frac{\partial}{\partial x}\left\{\left(\frac{\varepsilon_m}{\sigma_k}+\nu\right)\frac{\partial k}{\partial x}\right\}+\frac{\partial}{\partial y}\left\{\left(\frac{\varepsilon_m}{\sigma_k}+\nu\right)\frac{\partial k}{\partial y}\right\}+\frac{\partial}{\partial z}\left\{\left(\frac{\varepsilon_m}{\sigma_k}+\nu\right)\frac{\partial k}{\partial z}\right\}, \tag{4a}$$

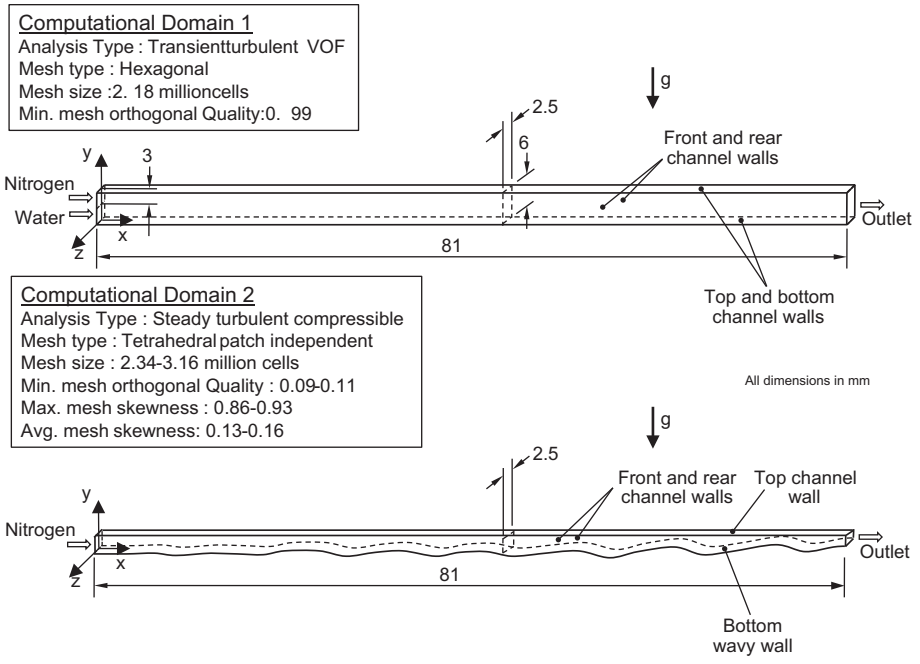


**Fig. 2.** Computational domains.

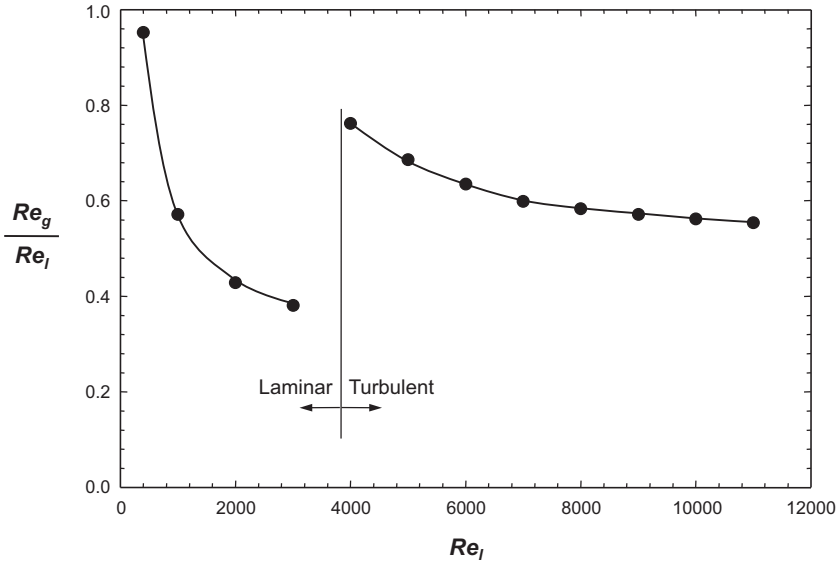


**Fig. 3.** Flow conditions for centerlined interface.

and

$$\frac{\partial \varepsilon}{\partial t}+\frac{\partial \varepsilon \bar{u}_x}{\partial x}+\frac{\partial \varepsilon \bar{u}_y}{\partial y}+\frac{\partial \varepsilon \bar{u}_z}{\partial z}=-C_{\varepsilon 1}\frac{\varepsilon}{k}\left(\langle u_x'^2\rangle\frac{\partial \bar{u}_x}{\partial x}+\langle u_x'u_y'\rangle\frac{\partial \bar{u}_x}{\partial y}+\langle u_x'u_z'\rangle\frac{\partial \bar{u}_x}{\partial z}+\langle u_y'u_x'\rangle\frac{\partial \bar{u}_y}{\partial x}+\langle u_y'^2\rangle\frac{\partial \bar{u}_y}{\partial y}+\langle u_y'u_z'\rangle\frac{\partial \bar{u}_y}{\partial z}\right)-C_{\varepsilon 1}\frac{\varepsilon}{k}\left(\langle u_z'u_x'\rangle\frac{\partial \bar{u}_z}{\partial x}+\langle u_z'u_y'\rangle\frac{\partial \bar{u}_z}{\partial y}+\langle u_z'^2\rangle\frac{\partial \bar{u}_z}{\partial z}\right)-C_{\varepsilon 2}\frac{\varepsilon^2}{k}+\frac{\partial}{\partial x}\left\{\left(\frac{\varepsilon_m}{\sigma_\varepsilon}+\nu\right)\frac{\partial \varepsilon}{\partial x}\right\}+\frac{\partial}{\partial y}\left\{\left(\frac{\varepsilon_m}{\sigma_\varepsilon}+\nu\right)\frac{\partial \varepsilon}{\partial y}\right\}+\frac{\partial}{\partial z}\left\{\left(\frac{\varepsilon_m}{\sigma_\varepsilon}+\nu\right)\frac{\partial \varepsilon}{\partial z}\right\}. \tag{4b}$$

Applying the Boussinesq eddy viscosity concept [29],

$$-\langle u_x'u_y'\rangle=\varepsilon_m\left(\frac{\partial \bar{u}_x}{\partial y}+\frac{\partial \bar{u}_y}{\partial x}\right), \tag{5a}$$

$$-\langle u_x'u_z'\rangle=\varepsilon_m\left(\frac{\partial \bar{u}_x}{\partial z}+\frac{\partial \bar{u}_z}{\partial x}\right), \tag{5b}$$

$$-\langle u_y'u_z'\rangle=\varepsilon_m\left(\frac{\partial \bar{u}_y}{\partial z}+\frac{\partial \bar{u}_z}{\partial y}\right), \tag{5c}$$

$$-\langle u_x'^2\rangle=2\varepsilon_m\frac{\partial \bar{u}_x}{\partial x}-\frac{2}{3}k, \tag{5d}$$

$$-\langle u_y'^2\rangle=2\varepsilon_m\frac{\partial \bar{u}_y}{\partial y}-\frac{2}{3}k, \tag{5e}$$

and

$$-\langle u_z'^2\rangle=2\varepsilon_m\frac{\partial \bar{u}_y}{\partial y}-\frac{2}{3}k, \tag{5f}$$

where

$$\varepsilon_m=C_\mu\frac{k^2}{\varepsilon}. \tag{6}$$

The present model utilizes standard values for all the constants in the above equations. For Domain 2, the $k$–$\omega$ model [30] is used because of its superior capability in addressing complex wall effects compared to the $k-\varepsilon$ model.

A recent study by the authors [3] points to the ability to extend FLUENT's flow modeling capability to tackle turbulent two-phase flows. In the present study, computations are performed using the FLUENT Analysis System in the Toolbox of ANSYS Workbench 14.0.0 [31]. The Project Schematic of Workbench in ANSYS FLUENT 14.5 is utilized to construct and mesh Domain 1. A User Defined Function (UDF) is written and applied to extract interfacial coordinates after steady state is attained, which are then rendered to form the volume that encompasses Domain 2. FLUENT is then used to perform the single-phase simulation for Domain 2. Overall, Domain 1 provides the interfacial profile and turbulent characteristics, and Domain 2 the interfacial stress and drag coefficient results.

At the interface in Domain 1, surface tension, molecular effects and gas shear effects are all considered. The model ensures that the tangential and normal force balance equations are always satisfied. To evaluate the local cell curvature, the volume fraction gradient is utilized in the continuum surface force model [32]. At the wall, wall adhesion in terms of the prescribed contact angle is taken to determine surface tension. The viscosity-influenced near-wall region is completely resolved all the way to the viscous sublayer, by demarcating this zone into a viscosity-influenced region and a fully turbulent region. In the fully turbulent region, the $k-\varepsilon$ model is employed to define turbulent viscosity, while the one-equation model of Wolfstein [33] is applied in the viscosity-influenced near-wall region; the length scale for turbulent viscosity in the latter is derived according to Chen and Patel [34]. The smooth blending of this multi-layer definition for turbulent viscosity with the high Reynolds number profile from the outer region is implemented as per [35]. Following [36], FLUENT then utilizes this two-layer model with a modified single function formulation of the law of the wall for the entire wall region, by blending laminar and turbulent law of the wall relations. This formulation ensures correct asymptotic behavior for large and small values of $y^+$ and suitable representation of velocity profiles where $y^+$ falls inside the buffer region.

For Domain 1, the two-phase treatment follows the Volume of Fluid (VOF) model [37]. In order to conserve computation time, the fractional step version of the non-iterative time advancement (NITA) scheme is used initially with first-order implicit discretization at every time step [38,39] to obtain pressure–velocity coupling. At convergence, third-order iterations are employed.

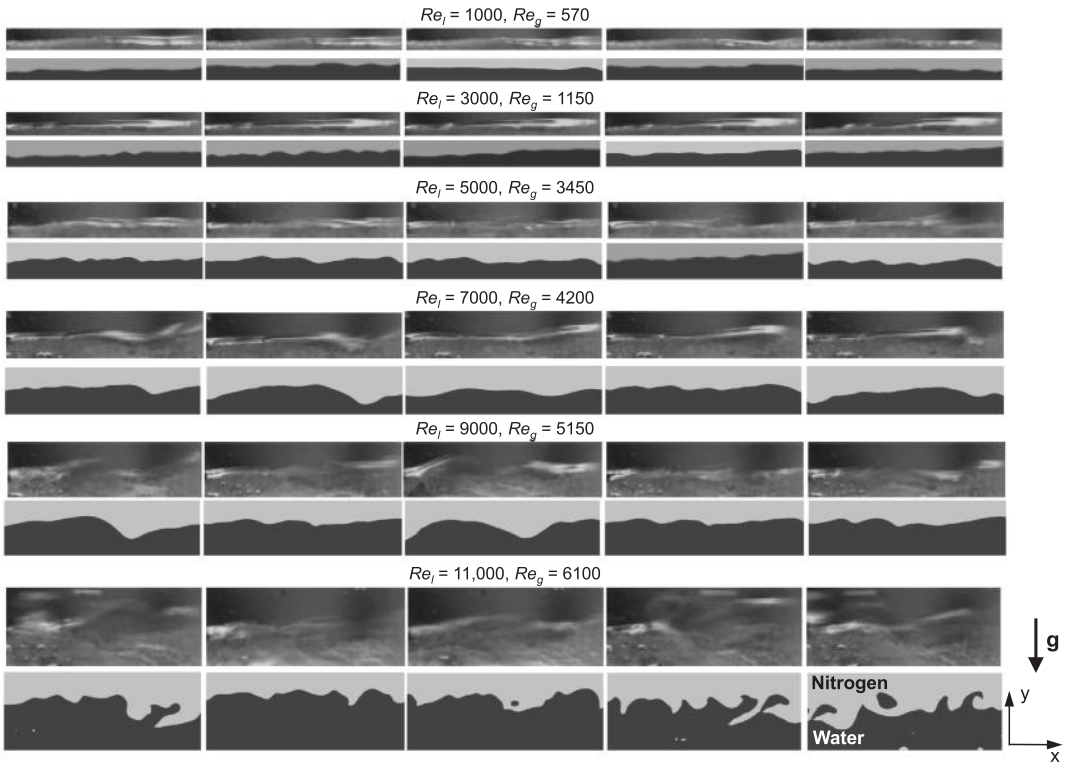


**Fig. 4.** Sequential images of water layer interface at $x = 54$ mm compared with corresponding computed interface plots for different water and nitrogen flow rates using Domain 1. Individual images in each sequence are 12-mm long and separated by 0.002 s.

Gradient generation during spatial discretization is accomplished using the least-squares cell-based scheme [40], while PRESTO, QUICK, Geo-reconstruct and first-order upwind schemes [41] are used for pressure, momentum, volume fraction and turbulent kinetic energy resolution, respectively.

Domain 2 is constructed as follows. The extracted interfacial coordinates are exported to a C-Sharp (C#) editor to collate the interfacial points in a readable format for commercial surface rendering software. MeshLab, product of Visual Computing Lab, is used to create the three-dimensional surface. Geomagic Studio, product of 3D Systems, is then used to remove surface inconsistencies. Afterwards, Blender 2.7 software is used to convert the surface into a volume by a sequential extrusion process that creates several internal self-intersections. Netfabb 5.0 is used to remove the self-intersections and supply the volume to Solidworks. In this final stage, the volume is converted to a format that can be recognized by ANSYS as a solid mesh. ANSYS FLUENT then performs a single-phase iterative steady state analysis on this volume.

Fig. 2 provides the grid size details for both domains, which are arrived at after careful assessment in pursuit of optimum degree of mesh refinement. This process involves evaluating the influence of mesh size on computational effort and quality of results. The grid system used is non-uniform, with a larger number of grid points near the walls and interface to achieve superior accuracy in resolving key flow parameters. It is important to note that the transition in refinement due to non-uniformity is gradual to avoid influencing the flow.

## 4. Flow visualization results

Fig. 4 shows the flow visualization results for a 12-mm segment of the flow channel centered at $x = 54$ mm for $Re_l = 1000$–$11{,}000$ and $Re_g = 570$–$6100$ per Fig. 3. Five sequential images, spaced 0.002 s apart, are presented for each combination of $Re_l$ and $Re_g$ to capture the transient behavior of the interface. Below each image is a corresponding computed interface plot. At $Re_l = 1000$ and $Re_g = 570$, the images and computed plots depict a relatively smooth interface with small amplitude waves. At $Re_l = 3000$ and $Re_g = 1150$, both the images and computed plots show an increase in the interfacial wave amplitude. At $Re_l = 5000$ and $Re_g = 3450$, the interface displays further increase in wave amplitude with a broader spectrum of wavelengths. For $Re_l = 7000$ and $Re_g = 4200$, and $Re_l = 9000$ and $Re_g = 5150$, there are further increases in wave amplitude along with nitrogen entrainment in the water layer. At $Re_l = 11{,}000$ and $Re_g = 6100$, the interface is marred by appreciable entrainment and deposition effects. The computed plots for this condition clearly capture the highly unstable interface and entrainment and deposition effects for this condition.

## 5. Computational results

### 5.1. Interfacial profile and Eddy diffusivity

Fig. 5 shows the shape of the interface at low, medium and high water Reynolds numbers. An inspection of these profiles is important, as they influence the degree of flow separation and frictional resistance exerted. As expected, an increase in flow rate causes the interface to be subjected to greater shear, but the input parameters maintain the sheared interface at the mid-plane. The interface is clearly three-dimensional with a discernable pattern in the $z$ direction. The disturbances are most prominent at the walls, and decrease in intensity towards the central plane, a consequence of the velocity gradient induced by the near wall effects.

The computed interfacial wave characteristics are examined in Fig. 6 with the aid of normalized probability density of the ratio of the wave height, $h$ to wavelength, $l$, for all $Re_l$ values considered. The normalized probability density, $p\ (h/l)$, is given by

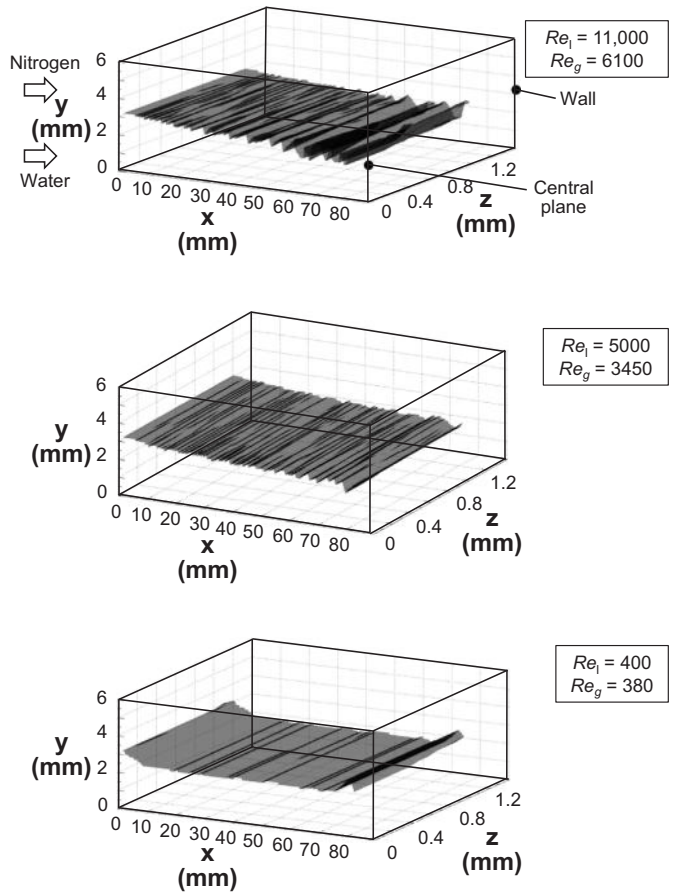


**Fig. 5.** Domain 1 computed interface plots for different flow rates.

$$p(h/l) = \frac{1}{\max\{p(h/l)\}}\left[\lim_{\Delta(h/l)\to 0}\frac{\mathrm{Prob}\{h/l < h/l(t) < h/l + \Delta(h/l)\}}{\Delta h/l}\right] = \frac{1}{\max\{p(h/l)\}}\frac{\mathrm{dP}(h/l)}{d(h/l)}, \tag{7}$$

where $P\,(h/l)$ is the probability distribution given by [42]

$$P(h/l) = \mathrm{Prob}\{h/l(t) < h/l\} = \frac{n\{h/l(t) < h/l\}}{N}. \tag{8}$$

Here, $P\,(h/l)$ describes the probability that the random variable, $h/l\,(t)$, will have a value less than or equal to a specified value, $h/l$, and $N$ and $n$ are the total number of time samples and number of time samples in a subset, respectively. The sampling is performed at $x = 54$ mm to ensure that the flow is unaffected by upstream or downstream effects. The total sampling time ranges from 0.1 s for $Re_l = 400$ to 0.01 s for $Re_l = 11{,}000$ in order to capture at least 10 waves.

Casting the interfacial characteristics in terms of probability density helps reduce the randomness in the data and eliminates noise due to any numerical instabilities. These statistically averaged characteristics are used later to develop relations for the interfacial stresses. Fig. 6 shows that the range of $h/l$ increases with increasing $Re_l$, especially for $Re_l = 400$–5000. This can be attributed to the trend captured in Fig. 3, where $Re_g/Re_l$ shows a significant decrease below $Re_l = 4000$. Notice in Fig. 6 that, because inertial effects are more balanced for $Re_l > 5000$, the increase in the $h/l$ range is far smaller than for $Re_l = 400$–5000.

Fig. 7 shows the time-averaged eddy diffusivity profiles across the water layer for different $Re_l$ and $Re_g$ values. Notice that the turbulence is completely suppressed at the interface because of surface tension effects as suggested in [1], and the eddy diffusivity profile has a broader span and different slope near the interface as compared to falling films [2]. It is also seen that, for most of the cases considered, peak eddy diffusivity is nearly constant, but the eddy diffusivity profiles are more distinct at high $y^+$ values.

### 5.2. Interfacial fluid dynamics

At $Re_l = 11{,}000$ and $Re_g = 6100$, the mean inlet velocities for water and nitrogen are 6.23 and 32 m/s, respectively, and for $Re_l = 400$ and $Re_g = 380$, the mean inlet velocities are 0.27 and 2.0 m/s, respectively. This shows that, for all cases considered, the mean velocity of nitrogen is appreciably greater than that of water. This explains the usefulness of Domain 2 computations, where the interface is assumed stationary to examine nitrogen flow behavior along the interface. Fig. 8 shows the $x$-velocity for

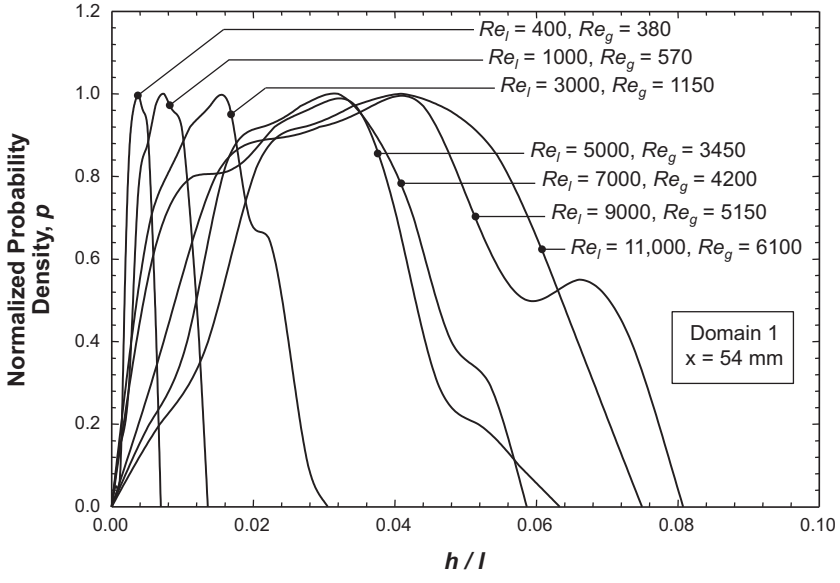


**Fig. 6.** Normalized probability density of $h/l$ for different $Re_l$ and $Re_g$ values.

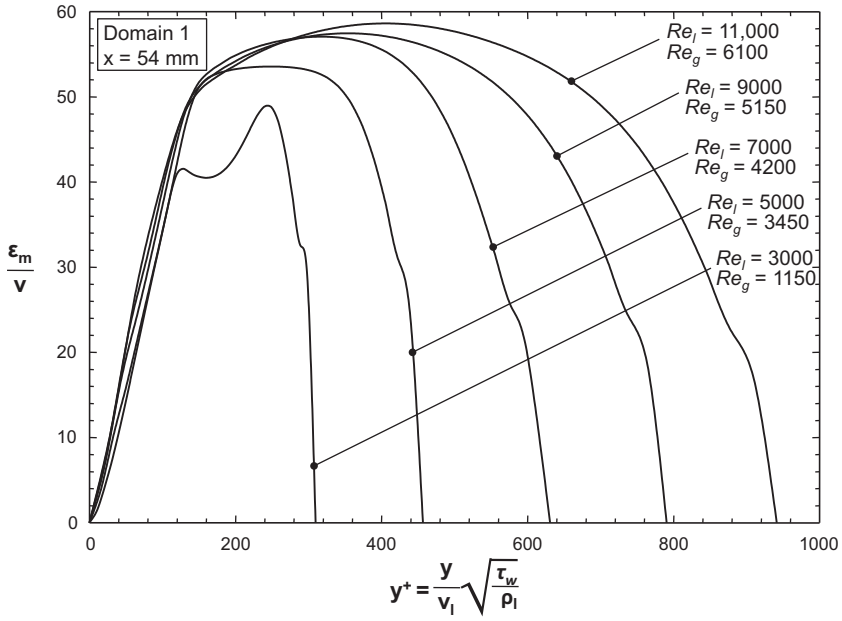


**Fig. 7.** Eddy diffusivity profiles across water layer for different $Re_l$ and $Re_g$ values.

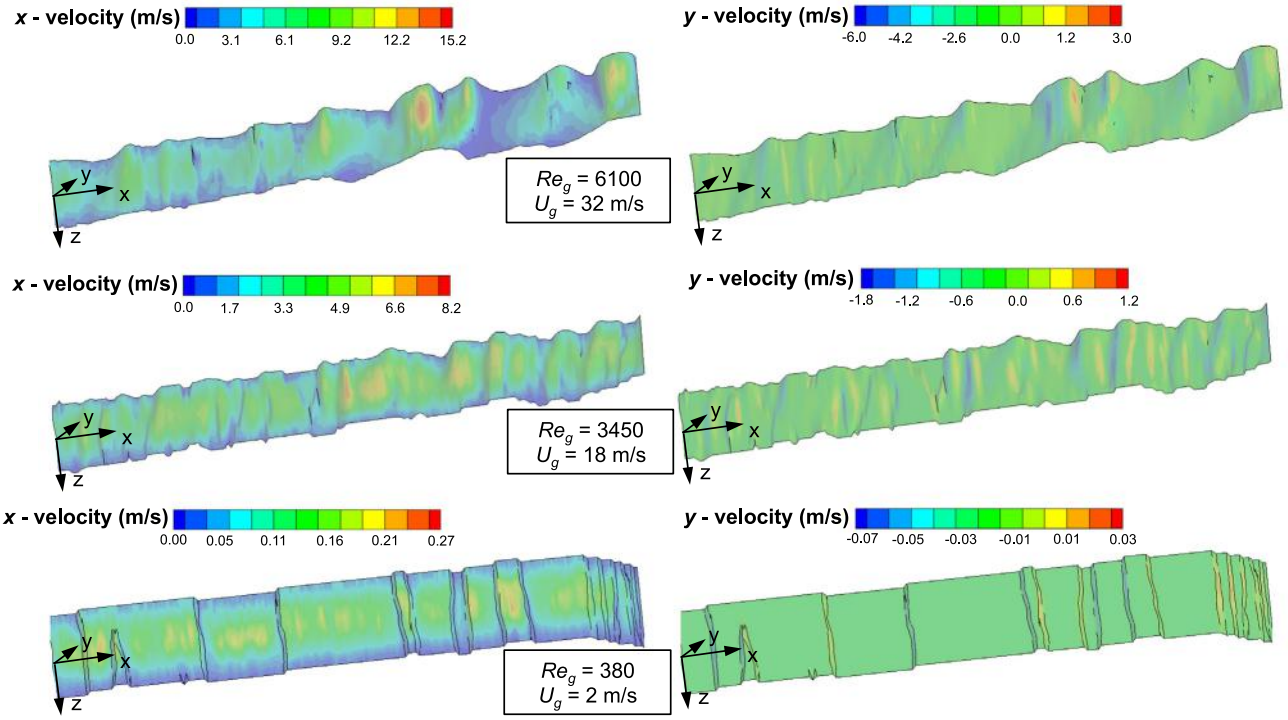


**Fig. 8.** Domain 2 (stationary interface) predictions of nitrogen *x*-velocity and *y*-velocity distributions along the interface for three operating conditions. The *y*-velocity is computed at 0.01 mm from the interface.

nitrogen flow along the curved but stationary interface (using Domain 2) at high, medium and low $Re_g$. Also shown is *y*-velocity at a small distance of 0.01 mm from the interface. For $Re_g = 6100$, the *x*-velocity is zero along the front and rear channel walls, highest along the crests of the large waves, and lowest in the deep troughs. For the same conditions, the *y*-velocity is expectedly appreciably smaller in magnitude than the *x*-velocity, and is highest in the advancing fronts of the large waves and lowest in the rear of the same waves. For $Re_g = 3450$, both the *x*-velocity and *y*-velocity are appreciably smaller than at $Re_g = 6100$, but show similar trends relative to the wave crests and troughs. At $Re_g = 380$, the *x*-velocity and *y*-velocity are comparatively quite small, and the interface far smoother than the other two cases, marred by few sharp curvature waves – ripples.

Fig. 9 shows Domain 2 nitrogen interfacial skin friction coefficient plots superimposed along the interface as well as projected from the interface for the same operating conditions as those in Fig. 8. The skin friction coefficient is defined as

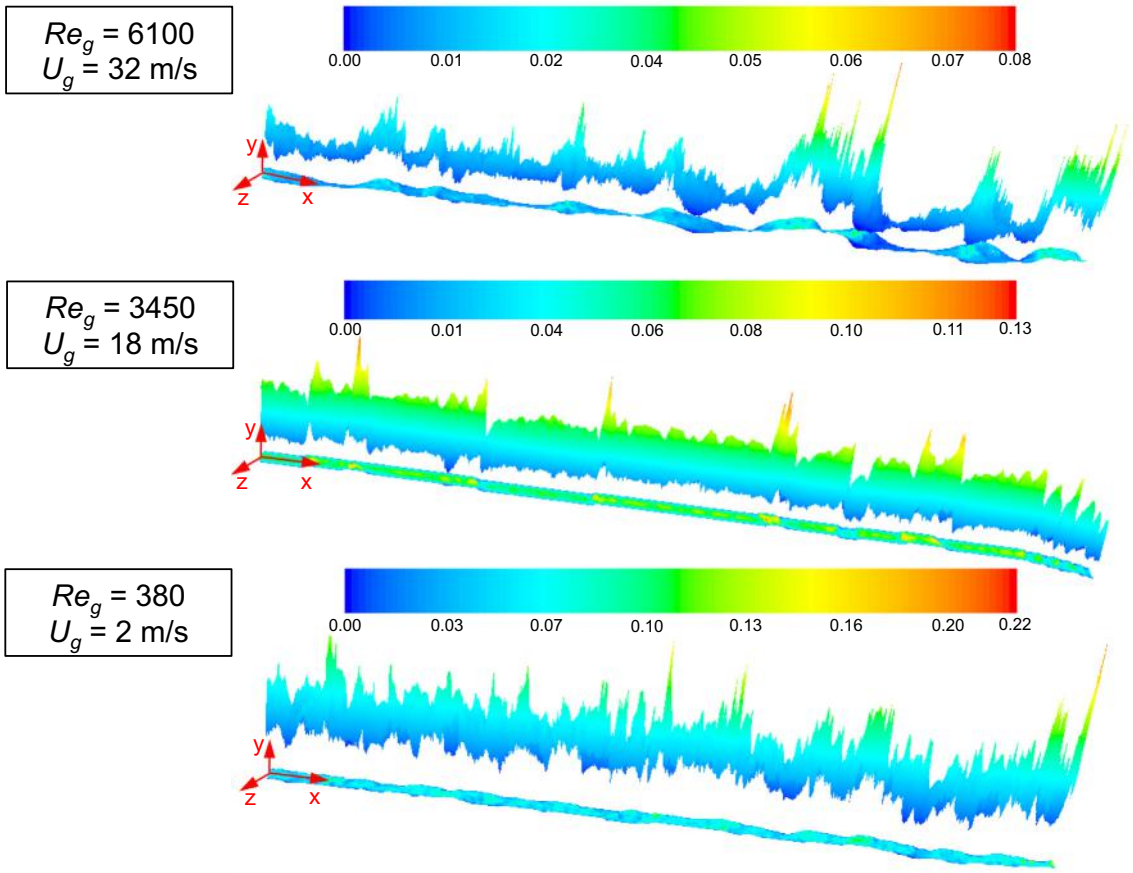


**Fig. 9.** Domain 2 (stationary interface) local nitrogen skin friction coefficient distributions, $C_{fi}$, for three operating conditions. Shown are skin friction coefficient plots superimposed along the interface as well as projected from the interface.

$$C_{f,i}(x,y,z) = \frac{\tau_i(x,y,z)}{\frac{1}{2}\rho_g u_{x,m}^2(x,z)}, \tag{9}$$

where $\tau_i$ is the interfacial shear and $u_{x,m}$ is the $x$-velocity at the corresponding $x$–$z$ location in Domain 2 averaged over the $y$ span, both computed using FLUENT. For $Re_g$ = 6100, the interface exhibits several large waves, which are associated with skin friction coefficient values up to 0.08. For $Re_g$ = 380, interfacial waviness is comparatively subdued but skin friction coefficient values are as large as 0.22.

Fig. 10 illustrates Domain 2 (stationary interface) computed separation effects in the nitrogen flow at highest and moderate $Re_g$ numbers captured in velocity vector and velocity contour plots and in a streamline plot for the highest $Re_g$. The velocity vector and velocity contour plots are presented for the central plane ($z$ = 0 mm). Notice in both the velocity vector and velocity contour plots, especially for $Re_g$ = 3450, how the nitrogen impinges against the tail (left side) of the middle interfacial wave, and separates along the front (right side) of the same wave. This produces a net drag force upon the interfacial wave in addition to the shear force. The plots for $Re_g$ = 6100 show appreciable swirl effects in the separation region, which is consistent with prior findings by the present authors concerning swirl effects in wavy, free-falling films [3]. These effects are also manifest in the streamline plot for $Re_g$ = 6100, where the nitrogen streamlines are shown departing from the interface downstream from the waves. To better appreciate separation effects, the present results are compared to a criterion by Stratford [43] for turbulent flow separation along a convex surface,

$$C_p'\sqrt{x'\frac{dC_p'}{dx}} \leqslant k\left(\frac{\mathrm{Re}_{x'}}{10^6}\right)^{0.1}, \tag{10}$$

where

$$C_p' = 1 - \left(\frac{\bar{u}_x}{\bar{u}_{x,\max}}\right)^2, \tag{11}$$

$$x' = \int_0^{l/2}\left(\frac{\bar{u}_x}{\bar{u}_{x,\max}}\right)^3 dx + x - l/2. \tag{12}$$

$\bar{u}_{x,\max}$ is the maximum $x$-velocity at the corresponding $x$-location along the central plane in Domain 2, $\bar{u}_x$ is the $x$-velocity at the corresponding $x$-location along the central plane in Domain 2 averaged over the $y$-span, $k$ = 0.35, and $l/2$ is the $x$-distance from the peak to the downstream edge of the surface. The surface in this criterion amounts to a single wave, however, the present study concerns a stationary interface with multiple waves. To apply his criterion to the present configuration, the coordinate $x$ in Eq. (12) is set to zero at the start of every interfacial wave and $\overline{\mu}_{x,max}$ (maximum velocity associated with the same wave) is computed from the FLUENT model. This criterion predicts that separation will not occur for $Re_g$ = 3450. However, as shown in Fig. 10, the present computations prove otherwise. It is not evident if the computed separation effects might be indication of the lack of applicability of the Stratford criterion for multiple waves, or the result of the $k$–$\omega$ model's tendency to generate excess separation [44].

### 5.3. Drag forces and coefficients

Fig. 11 shows Domain 2 (stationary interface) computed viscous and form drag forces for different $h/l$ values. Viscous drag is the force exerted by the nitrogen on the stationary interface due to

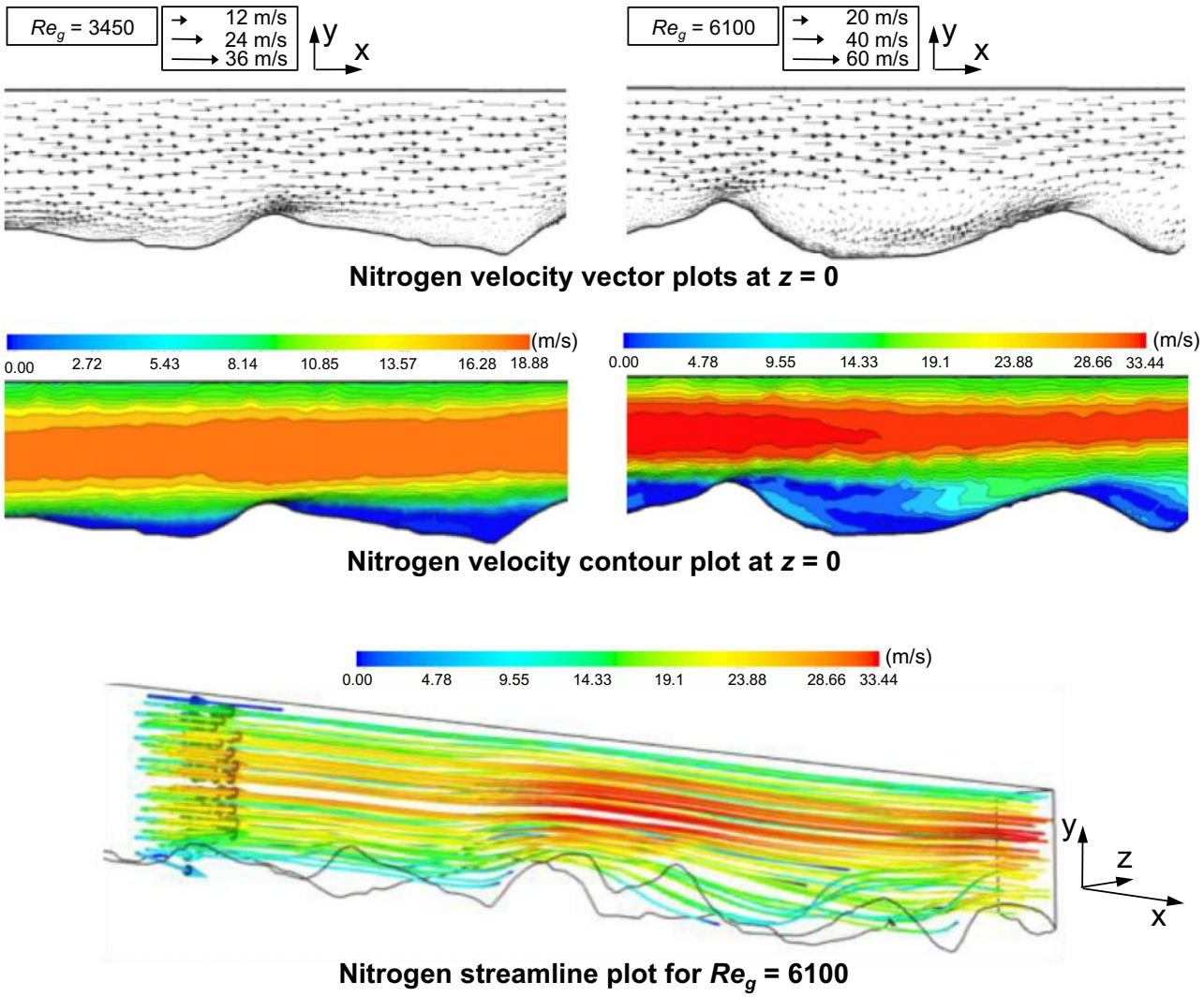


**Fig. 10.** Domain 2 (stationary interface) computed nitrogen velocity vector and contour plots along $z = 0$ (central plane) and $x = 54$ mm for two operating conditions, and streamline plot for highest $Re_g$ and same axial location.

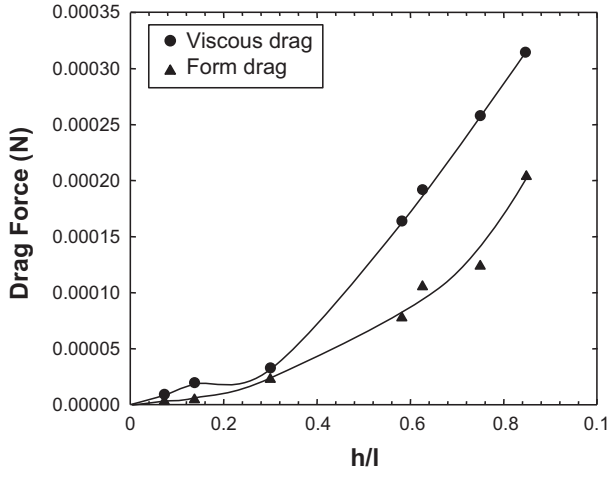


**Fig. 11.** Domain 2 (stationary interface) viscous and form drag force variations with $h/l$. The values of $h/l$ used in this plot correspond to the largest values obtained from Fig. 6 for different combinations of $Re_l$ and $Re_g$.

frictional resistance, and is calculated using the interfacial shear distribution as

$$F_{D,visc} = \int \tau_w \hat{t}.\hat{e}_d dS. \tag{13}$$

Here, $\tau_w$ is the local shear on the interface, $\hat{t}$ is the unit vector parallel to the interface, $\hat{e}_d$ is the unit vector parallel to the flow direction, and $dS$ is the differential area of the curved interface. Form drag is the inertial force caused by separation of the boundary layer from the interface and the wake created by the separation, calculated from the pressure distribution as

$$F_{D,form} = \int P\hat{n}.\hat{e}_d dS. \tag{14}$$

Here, $P$ is the local pressure and $\hat{n}$ the unit vector normal to the interface. The values of $h/l$ used in Fig. 11 correspond to the largest values obtained from Fig. 6 for different combinations of $Re_l$ and $Re_g$. The viscous and form drag forces are calculated by integrating the local drag components over the curved interfacial area, followed by dividing the integrated value over the area itself. Both forces

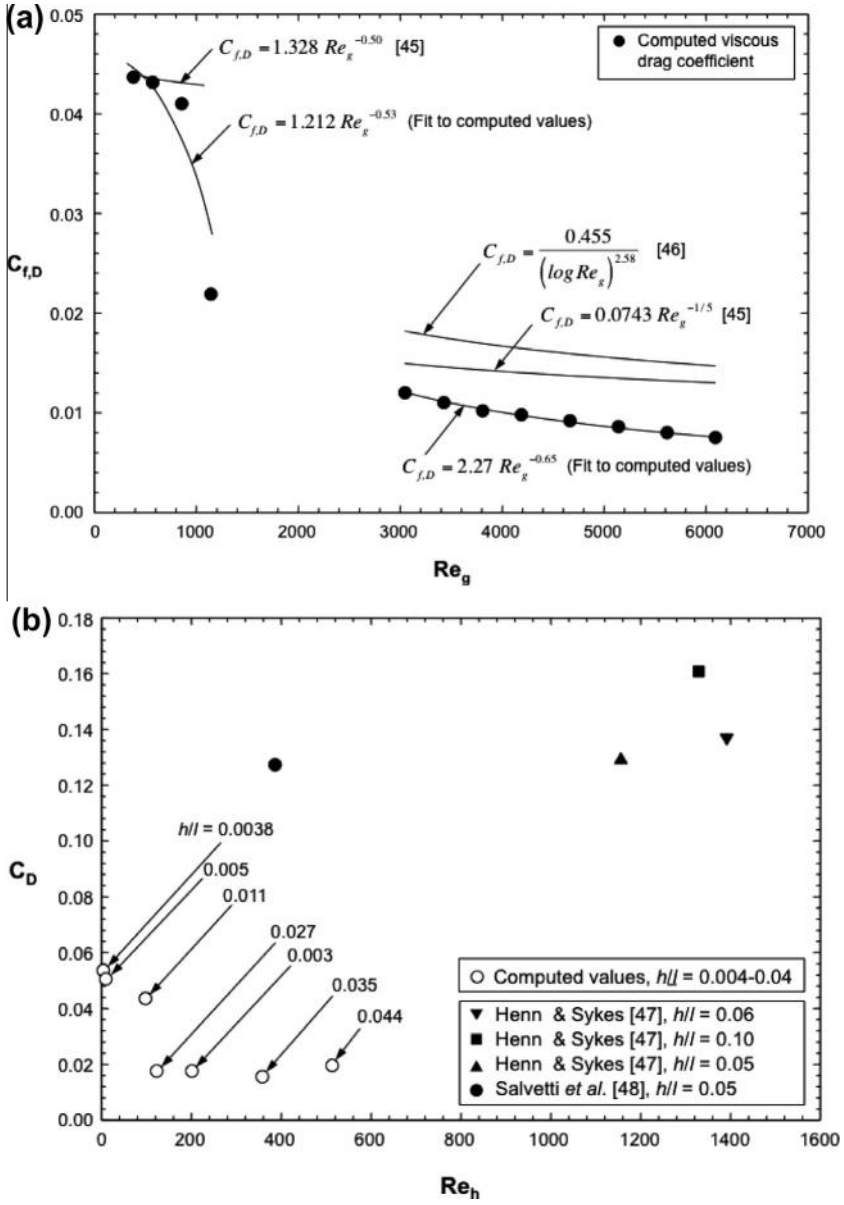


**Fig. 12.** (a) Computed Domain 2 (stationary interface) nitrogen viscous drag coefficient versus $Re_g$ compared to prior laminar and turbulent correlations for flow over a flat surface. (b) Computed Domain 2 nitrogen drag coefficient versus $Re_h$ compared to prior experimental correlations for flow over sinusoidal surfaces.

increase monotonically with increasing $h/l$. The viscous drag trend is attributed to the increase in wall friction since $h/l$ increases with increasing $Re_g$, i.e., with increasing flow velocity. The discontinuity in the viscous drag force plot around $h/l = 0.3$ can be attributed to the transition between laminar and turbulent flows as depicted in Fig. 3. The form drag increases with increased angle of attack; it is negligible for very small values of $h/l$ and becomes important for $h/l > 0.4$, where separation effects become significant.

Fig. 12(a) shows Domain 2 computed and fitted viscous drag coefficients versus $Re_g$, along with prior experimental correlations for viscous drag for flat surfaces. The viscous drag coefficient is a dimensionless measure of the viscous drag, defined as

$$C_{f,D} = \frac{F_{d,visc}/A}{\frac{1}{2}\rho_g U^2}, \tag{15}$$

where $A$ is the flat projected interfacial area and $U$ is the mean inlet $x$-direction velocity. The computed laminar and turbulent viscous drag friction coefficients are fitted according to the following relations:

$$\text{Laminar}: \quad C_{f,D} = 1.212\text{Re}_g^{-0.53}, \tag{16a}$$

$$\text{Turbulent}: \quad C_{f,D} = 2.27\text{Re}_g^{-0.65}. \tag{16b}$$

Overall, the inverse dependence of the viscous drag coefficient on $Re_g$ is consistent with the trends of prior correlations. There is better agreement in magnitude with correlations for the turbulent region [45,46]. The predictions do not compare well in the laminar region. The differences between computed and experimental trends can be attributed to the curvature effects, which are not accounted for in flat plate correlations.

Fig. 12(b) shows Domain 2 computed and fitted combined drag coefficients versus $Re_g$ along with prior experimental correlations for flow over sinusoidal surfaces [47,48]. The combined drag coefficient is a dimensionless measure of the combined effect of the viscous and form drag, defined as

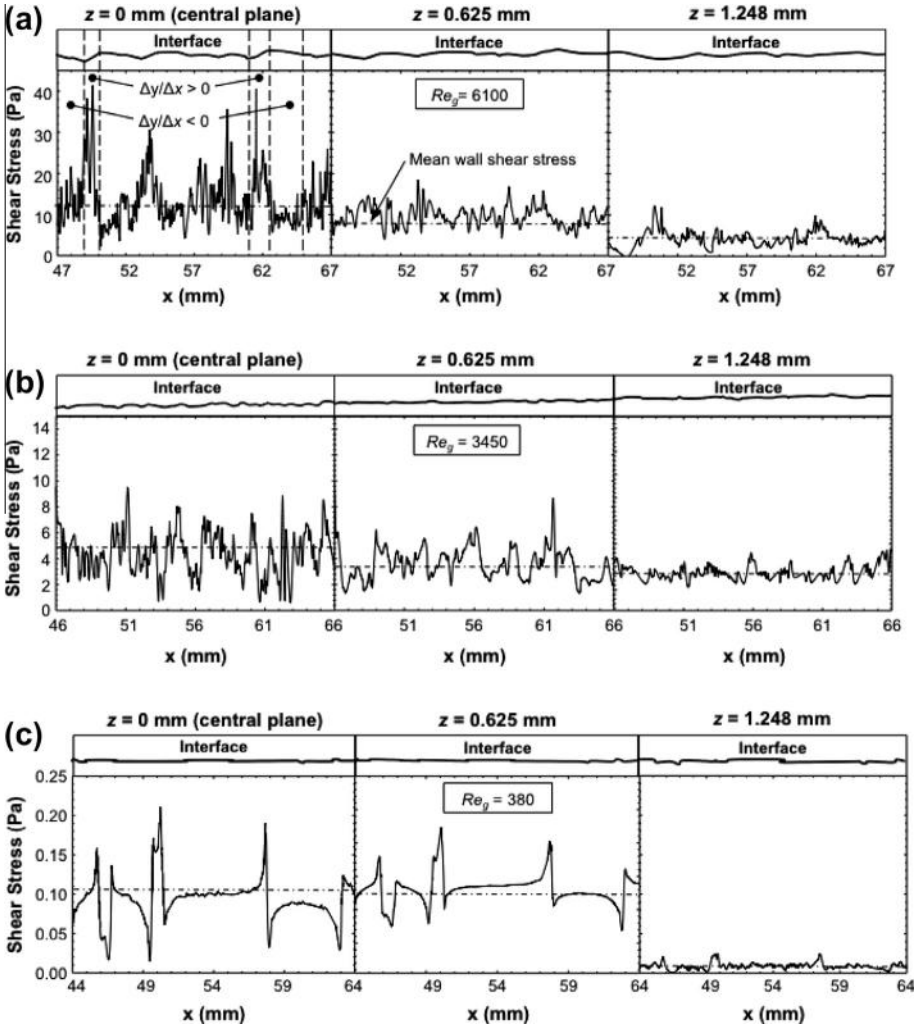


**Fig. 13.** Computed Domain 2 (stationary interface) nitrogen shear stress distributions for (a) $Re_g$ 6100, (b) $Re_g$ = 3450, and (c) $Re_g$ = 380.

$$C_D = \frac{F_D/A}{\frac{1}{2}\rho_g u_{x,m}^2}. \tag{17}$$

It is apparent that the predictions are well below those for sinusoidal surfaces. Differences can be attributed to several factors. First, the $h/l$ range considered is lower than $h/l$ available for comparison. Second, the $h/l$ range for the present conditions is not consistently maintained over the entire interface. Third, the present interfacial profiles are not sinusoidal, and their angle of attack is therefore considerably different.

### 5.4. Interfacial shear stress

Fig. 13(a)–(c) show distributions of interfacial shear stress for $Re_g$ = 6100, 3450 and 380, respectively, which accounts for viscous and form drag combined. Shown in each figure are the shear stress and interface profile at $z$ = 0 (central plane), 0.625 and 1.248 mm. For the case of $Re_g$ = 6100, where the interfacial undulation is pronounced, additional shear characteristics are highlighted. These plots show the shear stress decreases in the wave fronts, because of boundary layer separation effects, and increases in the wave tails because of stronger boundary layer attachment in these regions. In the presence of separation, these wave front and wave tail differences both increase form drag on the wave and viscous drag on the tail. These trends are not easily discernable at the lower flow rates, where the interfacial wave structure is less pronounced. Fig. 13(a)–(c) show the interfacial shear stress increases with increases in flow rate and/or distance from the wall. Notice in Fig. 13(c) the significant reduction in the magnitude of interfacial shear at $z$ = 1.248 mm compared to $z$ locations at or closer to the centerline. This can be explained by the thicker wall boundary layer at this low Reynolds number.

In order to derive an estimate of mean interfacial shear stress over the portion of the interface spanning the entire $z$-direction width and $x$-direction range indicated in Fig. 13(a)–(c), the local interfacial shear stress is integrated and averaged over the same area. For comparison, shear is calculated using the popular semi-empirical correlation by Wallis [6]

$$\tau_i(x,z) = \frac{1}{2} f_i \rho_g [u_{x,m}(x,z) - u_i(x,z)]^2. \tag{18}$$

For turbulent flow, the interfacial friction factor, $f_i$, is replaced by the expression [14]

$$f_{i,t} = f_g \left[ 1 + 300 \left\{ \left(\frac{\delta}{D}\right) - \frac{5}{\mathrm{Re}_g} \sqrt{\frac{2}{f_g}} \right\} \right], \tag{19}$$

where $\delta$ is the liquid film thickness, $D$ the hydraulic diameter of the channel, and $f_g$ the wall friction factor. $f_g$ in the laminar region is given by the Fanning friction factor,

$$f_{g,l} = \frac{14.227}{\mathrm{Re}_g}, \tag{20}$$

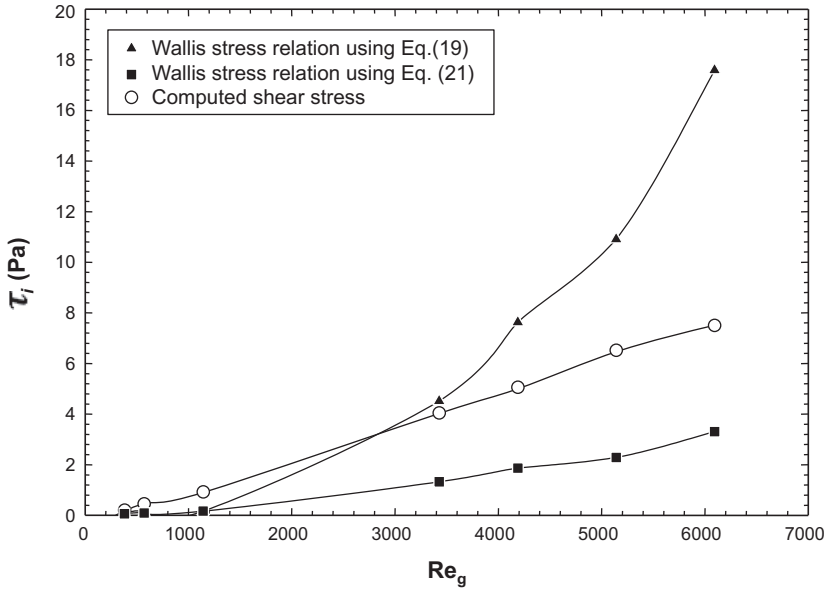


**Fig. 14.** Computed Domain 2 (stationary interface) nitrogen interfacial shear stress versus $Re_g$ compared to predictions of Wallis [6].

and in the turbulent region by the Blasius friction factor,

$$f_{g,t} = \frac{0.079}{\mathrm{Re}_g^{0.25}}. \tag{21}$$

For the laminar range, the friction factor $f_i$ in Eq. (18) is replaced by the Fanning friction factor, *i.e.*, $f_{i,l} = f_{g,l}$. Alternatively, there is also an established practice of using the Blasius wall friction factor instead of $f_{i,t}$, in Eq. (18) [49]. Fig. 14 shows the variation of the computed interfacial shear with $Re_g$ along with predictions of the Wallis correlation using $f_{i,t}$ and $f_{g,t}$. The computed interfacial shear increases fairly linearly with increasing $Re_g$. The Wallis correlation using $f_{i,t}$ overpredicts the computed values for high $Re_g$, but shows better agreement in the laminar and transition regions. A possible explanation for the departure between the computed and Wallis results is the inability of the Wallis function, $f_{i,t}$, to account for the changing character of interfacial waves with increasing $Re_g$. Fig. 14 shows the Wallis shear stress relation using $f_{g,t}$ underpredicts the computed results, but provides fair agreement in slope.

Kataoka et al. [10] studied interfacial shear in the presence of interfacial waves, and proposed that the Wallis friction factor can better represent interfacial shear by incorporating the wave height in Eq. (19). Using the same rationale, the computed data are fitted according to

$$f_{i,t} = f_g\left[1 + 300\left\{0.22\left(\frac{h}{l}\right)^{-0.4}\left(\frac{\delta}{D}\right) - \frac{5}{Re_g}\sqrt{\frac{2}{f_g}}\right\}\right]. \tag{22}$$

Fig. 15(a) shows the variation of computed friction factor with $Re_g$, along with the Fanning correlation and curve fit for the computed turbulent region using Eq. (22). The computed $f_i$ is arrived at by inserting the computed shear stress into Eq. (18).

From the literature, correlations for $f_{i,t}$ (for the turbulent region) were presented by Moeck [21], Fore et al. [26], Fukano and Furukawa [25], and Wongwises and Kongkiatwanitch [27], respectively, as

$$f_{i,t} = 0.005\left[1 + 1458\left\{\left(\frac{\delta}{D}\right) - \frac{5}{\mathrm{Re}_g}\sqrt{\frac{2}{f_g}}\right\}^{1.42}\right], \tag{23a}$$

$$f_{i,t} = 0.005\left[1 + 300\left\{\left(\frac{\delta}{D}\right) - \frac{5}{\mathrm{Re}_g}\sqrt{\frac{2}{f_g}} - 0.0015\right\}\right], \tag{23b}$$

$$f_{i,t} = 0.425\left(12 + \frac{v_f}{v_w}\right)^{-1.33}\left[1 + 12\left\{\left(\frac{\delta}{D}\right) - \frac{5}{\mathrm{Re}_g}\sqrt{\frac{2}{f_g}}\right\}\right]^8, \tag{23c}$$

and

$$f_{i,t} = 17.172\mathrm{Re}_g^{-0.768}\left\{\left(\frac{\delta}{D}\right) - \frac{5}{\mathrm{Re}_g}\sqrt{\frac{2}{f_g}}\right\}^{-0.253}. \tag{23d}$$

In Eq. (23c), $v_w$ and $v_g$ are the kinematic viscosities of water and nitrogen, respectively, at 20 °C.

Fig. 15(b) compares predictions of the above correlations with those using Eq. (19) for the turbulent region using the same $\delta/D$ ratio as the present Domain 1 configuration. For the laminar region, Eq. (20) is used in conjunction with both the present computations and all previous correlations. It is seen that [25] overpredicts the fitted data, an observation previously made in [27]. However, the fit proposed by [27] also shows significant departure in the medium flow rate range. The current fit agrees well with [14,21,26] for medium $Re_g$ values up to about 3500, where interfacial waviness is still relatively mild, but shows slight departure from the previous correlations for higher $Re_g$, where waviness is more pronounced.

The present findings point to the importance of conducting detailed interfacial measurements (liquid and gas layer thicknesses) in shear driven flows, along with velocity measurements to validate computational models. Such measurements have been conducted in adiabatic falling liquid films [50,51] and provide valuable insight into both turbulence structure and interfacial waviness, albeit for relatively thick liquid layers. As discussed in [52], measurements of liquid velocity in thin layers are possible with the aid of micro-particle image velocimetry (μ-PIV), but optical requirements palace stringent limits on the size and shape of the flow channel. Given the small thickness of shear-driven liquid layers, better and more miniaturized diagnostic tools are needed to measure liquid layer thickness and characterize interfacial waves [5,53,54], as well as temperature profile across the liquid layer [55,56] in non-adiabatic systems. Similar diagnostic tools can also play a vital role in understanding more complex two-phase phenomena, such as the formation of a wavy vapor–liquid wall layer that precedes the formation of critical heat flux (CHF) in flow boiling [57–60]. A unique aspect of the waves captured in CHF studies is unusually high $h/l$ values that exceed those of both the present

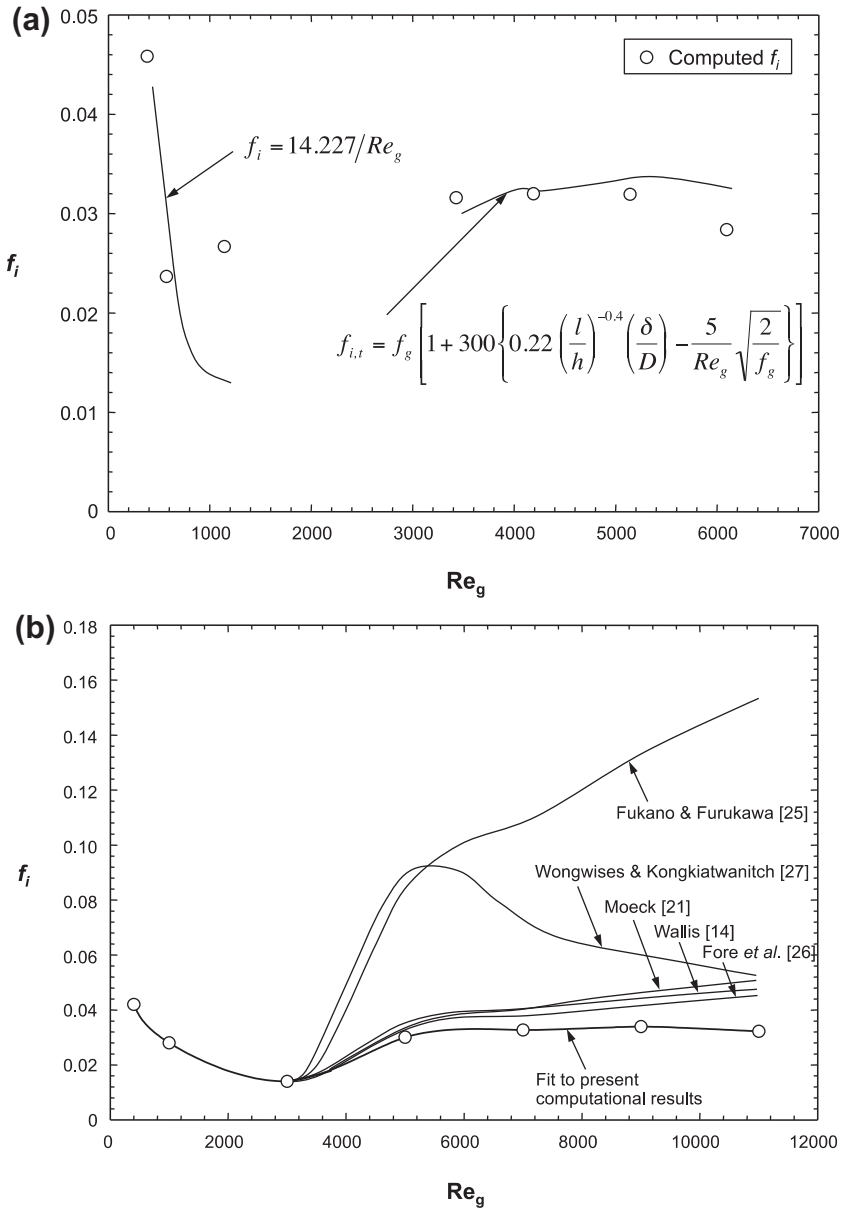


**Fig. 15.** (a) Variation of interfacial friction factor with nitrogen Reynolds number. (a) Curve fits to present computed results. (b) Comparison of present curve fit to previous correlations.

study and previous correlations and sinusoidal wall models. These high $h/l$ values contribute appreciable flow separation in the wave fronts and are therefore likely to yield unusually high interfacial shear.

## 6. Conclusions

This study examines the fluid dynamics of the wavy interface between horizontal liquid and gas layers using water and nitrogen as working fluids. A test facility is developed to capture the interfacial behavior using high speed video. Using FLUENT, computational models are developed for two distinct domains. Domain 1 is comprised of the actual two-phase flow, and used to explore interfacial structure and eddy diffusivity. Because the mean velocity of the gas flow is much greater than that of the liquid, the liquid layer serves essentially as a solid wavy wall as far as the gas flow is concerned. Domain 2 consists of the gas flow alone, with the actual interfacial boundary computed using the first domain replaced by a wavy stationary wall. Domain 2 is employed to explore flow separation effects around solid waves, as well as the three-dimensional variations of both viscous and form drag. The computational results are fitted to a function of both the gas Reynolds number, $Re_g$, and ratio of wave height to wavelength. Key findings from the study are as follows.

1. A consistent ratio of liquid to gas Reynolds numbers is required to maintain a fairly horizontal interface. Flow visualization reveals increased waviness and entrainment effects with increasing flow rates of the two fluids. These trends are captured well by FLUENT, albeit with reduced interfacial intensity.
2. The interface is marred by complex three-dimensional features. The range of ratios of wave height to wavelength grows appreciably up to a liquid Reynolds number of about 5000, above which the growth is much slower.
3. Computed results using Domain 1 show turbulence at the interface is completely suppressed by surface tension, and the time-averaged eddy momentum diffusivity profile across the liquid

layer has a broader span and different slope near the interface compared to eddy diffusivity across free falling films. These differences are attributed to the influence of interfacial shear for the present gas–liquid flow.

4. Computed results using Domain 2 show the mean gas velocity is appreciably greater than that of the liquid for all cases considered. The $x$-velocity is highest in the advancing fronts of large waves and lowest in the rear of the same waves. This trend becomes most pronounced with increasing $Re_g$.
5. Velocity vector plots, contour plots and flow streamlines computed using Domain 2 show interfacial flow separation effects on the gas side beyond $Re_g = 3450$, and these effects are amplified with increasing $Re_g$. The gas flow impinges against the tails of large wall waves, and separates over the wave fronts. This produces form drag along the wavy interface in addition to the viscous drag.
6. The interfacial viscous and form drag components increase monotonically with increasing ratio of wave height to wavelength because of the increased frictional resistance and flow separation effects. A new relation for the interfacial friction factor is derived from the computational results, which agrees well with prior turbulent flow correlations.

## Conflict of interest

None declared.

## Acknowledgment

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